EXERCICE№5

$U_n = \frac{2^n}{(-5)^{n+1}} = \frac{-1}{5}.\left(\frac{-2}{5}\right)^n$; $\lim\limits_{n\to+\infty} U_n = 0$, car $(-1 < \frac{-2}{5} < 1)$

EXERCICE№6

1) $U_n = n + \sin n$

$\sin n \geq -1 \implies n + \sin n \geq n - 1 \implies U_n \geq n - 1$ et $\lim\limits_{n\to+\infty}(n-1) = +\infty$

$\implies \lim\limits_{n\to+\infty} U_n = +\infty$

2) $U_n = \frac{1}{n}\left[\sin n + \left(\frac{1}{2}\right)^n\right]$; $(-1 \leq \sin n \leq 1)$

$\implies -1 + \left(\frac{1}{2}\right)^n \leq \sin n + \left(\frac{1}{2}\right)^n \leq 1 + \left(\frac{1}{2}\right)^n$

$\implies \frac{1}{n}\left[-1 + \left(\frac{1}{2}\right)^n\right] \leq U_n \leq \frac{1}{n}\left[1 + \left(\frac{1}{2}\right)^n\right]$

$\lim\limits_{n\to+\infty} \frac{1}{n}\left[-1 + \left(\frac{1}{2}\right)^n\right] = \lim\limits_{n\to+\infty} \frac{1}{n}\left[1 + \left(\frac{1}{2}\right)^n\right] = 0$ d'où $\lim\limits_{n\to+\infty} U_n = 0$

3) $U_n = -1 + \frac{1}{n^2}(\cos n + n)$; $(-1 \leq \cos n \leq 1)$

$\implies n - 1 \leq \cos n + n \leq n + 1 \implies \frac{1}{n} - \frac{1}{n^2} \leq \frac{1}{n^2}(\cos n + n) \leq \frac{1}{n} + \frac{1}{n^2}$

$\implies \frac{1}{n} - \frac{1}{n^2} - 1 \leq U_n \leq \frac{1}{n} + \frac{1}{n^2} - 1$

$\lim\limits_{n\to+\infty}\left(\frac{1}{n} - \frac{1}{n^2} - 1\right) = \lim\limits_{n\to+\infty}\left(\frac{1}{n} + \frac{1}{n^2} - 1\right) = -1$ d'où $\lim\limits_{n\to+\infty} U_n = -1$

EXERCICE№7

1) $U_n = \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\ldots\ldots\times\frac{n-1}{n}$ en simplifiant on aura :

$U_n = \frac{1}{n}$ et $\lim\limits_{n\to+\infty} U_n = 0$

2) $V_n = \frac{3}{2}\times\frac{4}{3}\times\frac{5}{4}\times\frac{6}{5}\times\ldots\ldots\times\frac{n+1}{n}$ en simplifiant on aura :

$V_n = \frac{n+1}{2}$ et $\lim\limits_{n\to+\infty} V_n = +\infty$

EXERCICE№8

$U_n = \left(\frac{2}{n}\right)^n$; $n \geq 1$

1) $n \geq 4 \implies \frac{1}{n} \leq \frac{1}{4} \implies \frac{2}{n} \leq \frac{1}{2}$ d'où $\left(\frac{2}{n}\right)^n \leq \left(\frac{1}{2}\right)^n \implies U_n \leq \left(\frac{1}{2}\right)^n$

2) $0 \leq U_n \leq \left(\frac{1}{2}\right)^n$ pour $n \geq 4$

$\lim\limits_{n\to+\infty}\left(\frac{1}{2}\right)^n = 0$ d'où $\lim\limits_{n\to+\infty} U_n = 0$

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